1.0

4.5
5.0
5.6

2.8

2.5

3.2

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A NUMERICAL HINDCAST MODEL FOR WAVE SPECTRA ON WATER BODIES WITH IRREGULAR SHORELINE GEOMETRY

Report I

TEST OF NONDIMENSIONAL GROWTH RATES

by

Donald T. Resio, C. Linwood Vincent

Hydraulics Laboratory
U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

August 1977
Report I of a Series

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A numerical model for wave generation in water bodies of arbitrary shape is described. The new model is a variation of a model previously developed by Barnett (1968). Important differences are the numerical technique, slightly modified source terms and the incorporation of a variable equilibrium range of spectral density. Results for ideal, homogeneous generation situations for growth with nondimensional time and nondimensional duration are examined and compared to those from some previous models.

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20. ABSTRACT (Continued).

These results indicate that growth with fetch in the present model is much closer than those of previous numerical models to the observed in recent field experiments. The growth with nondimensional time is much more difficult to assess, but the proposed model appears to give reasonable results for these situations as well. Report 2 of this series will document an extensive set of real-time tests of the new model at several gage sites on the Great Lakes.

PREFACE

A request for the U. S. Army Engineer Waterways Experiment Station (WES) to conduct an investigation of wave heights on the Great Lakes was made by the U. S. Army Engineer Division, North Central (NCD), in a conference held in Chicago, Illinois, on 22 July 1974. Funds were authorized by NCD on 30 August 1974. The study was conducted during the period from September 1974 to October 1976 in the Coastal Branch, Wave Dynamics Division, Hydraulics Laboratory, WES, under the direction of Mr. H. B. Simmons, Chief of the Hydraulics Laboratory, and Dr. R. W. Whalin, Chief of the Wave Dynamics Division.

Drs. D. T. Resio and C. L. Vincent conducted the study and also prepared the report. Mrs. Rebecca Brooks was especially helpful in performing analytical and programming tasks.

A special acknowledgement is due to Drs. D. Lee Harris (Coastal Engineering Research Center), Tim Barnett (Scripps Institution of Oceanography), and Vincent Cardone (City University of New York) for their review and constructive comments on the text of this report.

Directors of WES during the conduct of the study and the preparation of this report were COL G. H. Hilt, CE, and COL J. L. Cannon, CE. Technical Director was Mr. F. R. Brown.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) AND
METRIC (SI) TO U. S. CUSTOMARY UNITS OF MEASUREMENT

Units of measurement used in this report can be converted as follows:

Multiply	By	To Obtain
U. S. Customary to Metric (SI)		
feet	0.3048	metres
miles (U. S. statute)	1.609344	kilometres
miles (U. S. nautical)	1.852	kilometres
miles per hour	1.609344	kilometres per hour
knots (international)	0.5144444	metres per second
square feet per second	0.09290304	square metres per second
degrees (angle)	0.01745329	radians
Metric (SI) to U. S. Customary		
centimetres	0.3937	inches
metres	3.28084	feet
square metres per second	10.7638	square feet per second
Celsius degrees or Kelvins	9/5	Fahrenheit degrees*

* To obtain Fahrenheit (F) temperature readings from Celsius (C) readings, use the following formula: $F = 9/5(C) + 32$. To obtain Fahrenheit readings from Kelvins (K), use: $F = 9/5(K - 273.15) + 32$.

A NUMERICAL HINDCAST MODEL FOR WAVE SPECTRA ON WATER
BODIES WITH IRREGULAR SHORELINE GEOMETRY

REPORT 1:   TEST OF NONDIMENSIONAL GROWTH RATES

PART I: INTRODUCTION

Although many experiments of wave growth have been made under
conditions of simple shoreline geometries under quasi-homogeneous wind
fields, there has been a lack of concerted effort to extend these consider-
ations to situations of wave generation by variable winds in areas
bounded by irregular shorelines. Pragmatic definitions of effective fetch
and effective duration have been used to estimate the effects of variations
in fetch and duration; however, there has been little effort to verify these
estimates.

This paper presents a description of a numerical model which solves
the two-dimensional (frequency-propagation direction) radiative transfer
equation. The model is capable of calculating wave growth and decay
within any arbitrarily-shaped body of water, provided that refraction
and diffraction are negligible at the scales of interest. It is
hypothesized that the integration of the two-dimensional source terms
and gradient fields will lead to approximately correct estimates of
wave growth under complex conditions.

This is the first of two reports and will attempt to demonstrate
that the model gives accurate estimates of nondimensional growth character-
istics in regions of simple geometry. The second report will present
an extensive set of comparisons between hindcast wave heights and recorded
wave heights for several sites in the Great Lakes. Since no effort
is made to consider only those waves generated along simple fetches
with constant winds, the second report provides a good evaluation of
the operational characteristics of the model under diverse fetch geometries
and variable wind speeds.

Previous numerical models for wave hindcasting have been developed
primarily for open ocean application. Inoue (1967) incorporated into a
model the theoretical works of Miles (1957, 1960) and Phillips (1957, 1966)
to give a plausible explanation of wave generation. In this model, Inoue

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used a spectral form proposed by Pierson and Moskowitz (1964) to limit the growth of the wave spectra. Cardone (1969) extended this model to include the effects of non-neutral stability on wave growth and provided a means of obtaining objective wind fields over the ocean on an operational basis.

Numerical models based on the works of Inoue, Cardone, and Pierson have been programmed at Navy Fleet Numerical Weather Central in Monterey, California. Comparisons between hindcasts with this model of wave growth and observed waves have indicated that the model appears to work well in reproducing wave heights at oceanic sites (Inoue, 1967; Bunting, 1970; Salfi, 1974; Lazanoff and Stevenson, 1975) and in large semi-enclosed seas (Lazanoff et al., 1973). In this paper, we shall follow the precedent of Dexter (1974) and refer to this model as the Inoue-Bunting (I-B) model.

Although the development of the wave spectrum in the I-B model appears to be adequate for moderate to strong winds over a large fetch, the model has not been thoroughly tested at short fetches (less than 100 miles). It is in this region that the magnitude of the Miles-Phillips growth mechanisms has been questioned by several recent studies. Hasselmann (1962, 1963a, 1963b), in a series of papers in the 1960's, presented strong theoretical grounds for the role of wave-wave interactions in the development of a wave spectrum. Much of his theoretical work has been supported by subsequent field and laboratory studies (Snodgrass et al., 1965; Barnett and Sutherland, 1968; Sutherland, 1968; Mitsuyasu, 1968a; Hasselmann et al., 1973). A conclusion drawn from careful measurements of wave spectra taken during the Joint North Sea Wave Project (JONSWAP) is that the shape of the wave spectrum and the rapid rate of growth of waves on the steep forward face of the spectrum is primarily due to the effects of wave-wave interactions (Hasselmann et al., 1973).

Barnett (1968) formulated a model which included a parameterized version of the wave-wave interactions derived by Hasselmann. A few comparisons have been made between hindcasts by the I-B and Barnett models (Dexter, 1974). However, there have not been sufficient tests against observed wave heights to conclusively favor either model, at least in terms of the single parameter, significant wave height.

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Little has been done to date to establish the reliability of these models in areas of complex shoreline configurations and short fetches. Such conditions exist in many coastal areas and inland bodies of water in the United States. Consequently, there is an acute need for a model calibrated under these conditions. Considerable field data and laboratory data (Sutherland, 1968, Mitsuyasu, 1968b, 1973, Ross et al., 1970; Schule et al., 1971; Hasselmann et al., 1973) are now available which indicate that the development of wave spectra with fetch is in general accordance with the similarity theory of Kitaigorodskii (1962). The work of Kitaigorodskii (1962) follows the general pattern between nondimensional wave and fetch characteristics established by researchers during World War II, Sverdrup and Munk (1947). As shown in Figure 1, the nondimensional growth curves estimated from the I-B model and the Barnett model are not in good agreement with the growth curves expected on the basis of these field and laboratory studies. The I-B model tested here uses the form for Phillips resonance as given by Cardone (1969). This term has since been modified somewhat (Salfi, 1974); however, since the overall rate of growth with fetch is dominated by the Miles' instability mechanism in this model, the results should be very similar to those given in Figure 1. The purpose of this paper is to establish a numerical model which can function in areas of complex shore-line geometry and is in good agreement with the growth rates determined by the Mitsuyasu (1968b) and Hasselmann et al. (1973) studies as presented in Figure 1.

In addition to the I-B and Barnett models, which essentially simulate the growth and decay of waves with theoretical and empirical energy exchange processes, Hasselmann et al. (1976) have formulated a parametric model of wave growth in which the shape of the spectrum is considered invariant. In this model the spectrum is viewed as containing a rapidly-responding range of frequencies up to the frequency of the spectral peak and a slowly progressing, steep forward face. This model was published after the work in this paper was performed and hence is omitted from comparisons of source terms and details of the model; however, some discussion of the Hasselmann et al. model will be included
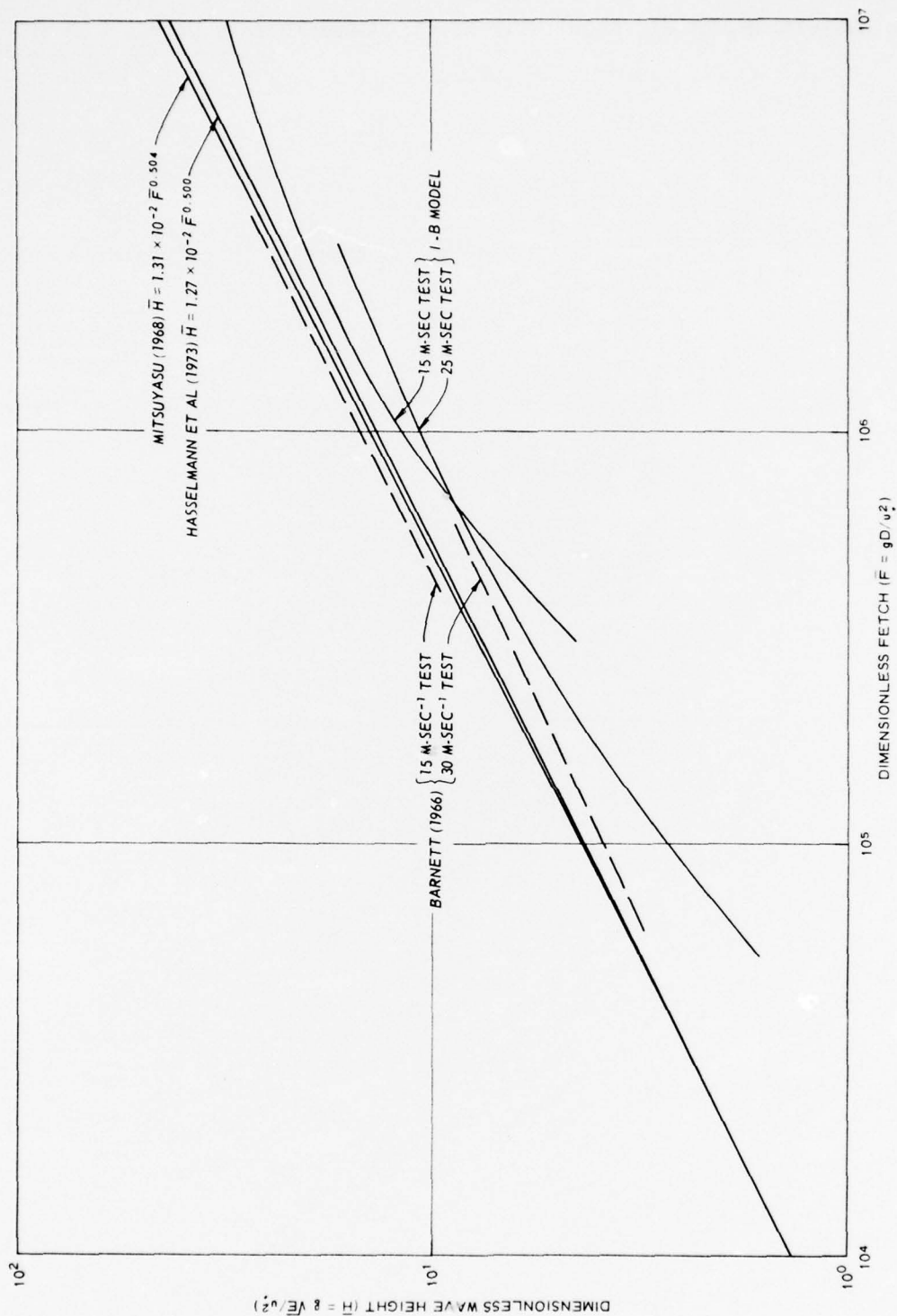
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Figure 1. Comparison of relationships between nondimensional wave height and nondimensional fetch to those derived empirically by Mitsuyasu (1968b) and Hasselmann et al. (1973)

DIMENSIONLESS FETCH ($\bar{F} = gD/u_*^2$)

DIMENSIONLESS WAVE HEIGHT ($\bar{H} = g\sqrt{\bar{E}}/u_*^2$)

MITSUYASU (1968) $\bar{H} = 1.31 \times 10^{-2} \bar{F}^{0.504}$

HASSELMANN ET AL (1973) $\bar{H} = 1.27 \times 10^{-2} \bar{F}^{0.500}$

$\left.\begin{array}{l}\text{15 M·SEC TEST}\\ \text{25 M·SEC TEST}\end{array}\right\}$ I·B MODEL

BARNETT (1966) $\left\{\begin{array}{l}\text{15 M·SEC}^{-1}\text{ TEST}\\ \text{30 M·SEC}^{-1}\text{ TEST}\end{array}\right.$

10^2

10^1

10^0

10^4

10^5

10^6

10^7

7

where approriate. Sanders (1976) presented a similar model; however,
only the Hasselmann, et al. model will be discussed in this paper.

PART II: THEORETICAL DEVELOPMENT

Given a two-dimensional wave number spectrum $F'(\underline{x},\underline{k},t)$ at location $\underline{x}$ and time t, it follows from the chain rule for partial derivatives that the rate of change of F' can be expressed as

$$\frac{DF}{Dt} = \frac{\partial F'}{\partial t} + \dot{x}_i \frac{\partial F'}{\partial x_i} + \dot{k}_i \frac{\partial F'}{\partial K_i} \qquad (1)$$

where the dot over the x_i and k_i symbolizes differentiation with respect to time and $\underline{k}$ is the wave number vector. The first term on the right side of Eq. 1 represents local sources and sinks of energy. The second and third terms represent advection of wave energy in $\underline{x}$-space and $\underline{k}$-space, respectively.

For computational simplicity, let us assume that the slope of the local bathymetry is everywhere sufficiently small so that the magnitude of the last term (primarily the effects of shoaling and refraction) is negligible compared to the first two terms. In a frame of reference moving along with a wave particle at its group velocity, Eq. 1 can then be redefined as

$$\frac{DF'}{Dt} = \frac{\partial F'}{\partial t} = S(\underline{x},\underline{k},t) \quad , \qquad (2)$$

where F' also is $F'(x,k,t)$. In the context of this paper, a wave particle is defined as a unit surface area containing energy density at a specified frequency and moving along with the propagation of this energy. S is the sum of all sources and sinks of wave energy and is a function of location, wave number, and time. A change to a frequency-direction representation of F can be made by the transformation

$$F(f,\theta) = \frac{2\pi k}{c_g} F' (k_1, k_2) \qquad (3)$$

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where k_1 and k_2 are orthogonal components of $\underline{k}$, c_g is the group velocity of waves with wave number modulus k given by

$$k = (k_1^2 + k_2^2)^{1/2} ,$$ (4)

f is the frequency of waves with wave number modulus k and θ is the direction of propagation defined by

$$\theta = \tan^{-1} (k_2/k_1) .$$ (5)

Since the group velocity is considered invariant in this case, Eq. 1 can be written in integral form as

$$F(x_1 + \varepsilon_1, x_2 + \varepsilon_2, f, \theta, t + \Delta t) = F(x_1, x_2 f, \theta, t)$$
$$+ \int_{t}^{t+\Delta t} S(\underline{x}(t), f, \theta, t) \, dt ,$$ (6)

where x_1 and x_2 are the orthogonal components of $\underline{x}$ and where ε_1 and ε_2 are given by

$$\varepsilon_i = \int_{t}^{t+\Delta t} \dot{x}_i \, dt = \dot{x}_i \, \Delta t .$$ (7)

For a sufficiently small time increment, Δt, the source function may be considered homogeneous over the propagation path and the integral on the right side of Eq. 6 may be approximated as

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$$\int_t^{t+\Delta t} S(x_1,x_2,t)\ dt = \int_t^{t+\Delta t} S(\underline{x}(t),t)\ dt \qquad (8)$$

for any wave particle with propagation direction and wave frequency (f,θ). This same approximation in a fixed frame of reference gives

$$F(x_1,x_2,t+\Delta t) = F(x_1,x_2,t) + \int S(x_1,x_2,t)\ dt$$

$$- \int_t^{t+\Delta t} \underline{\dot{x}} \cdot \underline{\nabla F}(x_1,x_2,t)\ dt \quad . \qquad (10)$$

Rearranging Eq. 9 and combining it with Eq. 6 permits an estimate of the energy variation over a small distance in $\underline{x}$-space at time $t+\Delta t$

$$F(x_1 x_2, t + \Delta t) = F(x_1 + \varepsilon_1, x_2 + \varepsilon_2, t + \Delta t)$$

$$- \int_t^{t+\Delta t} \underline{\dot{x}} \cdot \underline{\nabla F}(x_1,x_2,t)\ dt \quad . \qquad (10)$$

However, in order for Eq. 10 to be exact, the gradient term must be constant over a distance equal to at least twice the propagation distance over time increment Δt. The degree of approximation involved in actual applications of Eq. 10 will be discussed in a later section.

The basic concept behind the numerical application of Eqs. 6 and 10 to wave hindcasting is to integrate the source terms along a set of wave particle paths which terminate at a grid point at the end of a time step. These separate integrations along the assumed uncoupled paths produces an estimate at the grid point at this time. Since the group velocity is specified to be a constant, the particle paths are constrained to originate somewhere on a circle of radius $c_g \Delta t$ and

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propagate normal to the circle inward to the grid point. A one-dimensional spectrum is then just the summation of all particles which originate on such a circle and terminate at the grid point. The initial value of F at the circle must be specified. After that, Eq. 10 may be used to develop an estimate of F on the circle which is coupled to the development of F at the grid point and the gradient field.

From Eqs. 6 and 10 with a grid point specified at location $i(=x_1 + \varepsilon_1)$, $j(=x_2 + \varepsilon_2)$ an approximation to the value of F at the $n + 1^{th}$ time step for frequency k and propagation direction ℓ can be written in the following form

$$F_{ijk\ell}^{n+1} = F_{ijk\ell}^{n} - (F_{ijk\ell}^{n} - F_{i+\delta_1,j+\delta_2,k,\ell}^{n} + F_{i,j,k,\ell}^{n-1} - F_{i+\delta_1,j+\delta_2,k,\ell}^{n-1}) \cdot$$

$$\Delta t / 2\Delta x_{\underline{\delta}} + R_{\underline{\delta}}(F_{i+\lambda_1,j+\lambda_2,k,\ell}^{n} - F_{i+\lambda_1',j+\lambda_2',k,\ell}^{n})/\sqrt{2} \; \Delta x \tag{11}$$

$$+ \sum_{\nu=1}^{P} S_{\nu}$$

with $\quad \underline{\delta} = \underline{\delta}(\ell)$

$\quad \underline{\lambda} = \underline{\lambda}(\ell)$

$\quad \underline{\lambda}' = \underline{\lambda}'(\ell)$

To maintain convenience of notation here and to be consistent with previous notations by others, k is used in this paper to denote either a wave number or a frequency counter. The distinction between the two uses can be easily obtained from their context. In Eq. 11 superscripts denote the time step and subscripts denote the location of the wave particle in space (i,j), its associated frequency (k) and its direction of propagation (ℓ). The terms with Δx characterize

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the grid spacing; and the role of the additional subscripts related to $\underline{\delta}, \underline{\lambda}$ and $\underline{\lambda}'$ is to add or subtract from the i,j coordinate in order to locate reference values for F. The function R is used to provide an interpolation between grid points and the S terms represent sources and sinks of wave energy.

The first term of the right hand side of Eq. 11 is the energy density associated with the particle located at grid point (i,j) in $\underline{x}$-space and point (k,ℓ) in $\underline{k}$-space. As seen in Eq. 10, the estimation of the initial value of F at a distance $c_g \Delta t$ back into the direction of propagation requires that the gradient term be subtracted from the value at (i,j). Since the form of the gradient term is an integral through time, the initial value (n-1th time step) and final value (nth time step) are averaged. The subscript δ on the Δx allows the gradient to be computed either along an axis or a diagonal. If the angle increment $\Delta\theta$ is smaller than $\pi/4$ or not a multiple of $\pi/4$, then the third term, $R_{\underline{\lambda}}$ is required to account for the gradient of F_ℓ normal to the direction of propagation. The value of R can be approximated for computational purposes in the following manner. First, as shown in Figure 2, the grid cell in which the wave particle originates is determined. The gradient between those corners of the grid cell not on the diagonal drawn through point (i,j) can be linearized and used to provide an adjustment to the value of F for wave particles not on an axis or a diagonal.

From Eq. 11, the value of F at time step n+1 is a function of the source terms and previous values of F as given in time steps n and n-1. Using the source terms defined by Barnett (1968) with minor modifications (Resio and Vincent, 1976) the rate of change along the propagation path is given by

$$\frac{dF}{dt} = \alpha + \Gamma + (\beta - \tau - \phi)F \quad , \tag{12}$$

where α and β represent sources of energy input from the winds similar to those proposed by Phillips (1957, 1966) and Miles (1957, 1960), respectively; Γ and τ represent the effects of wave-wave interactions as
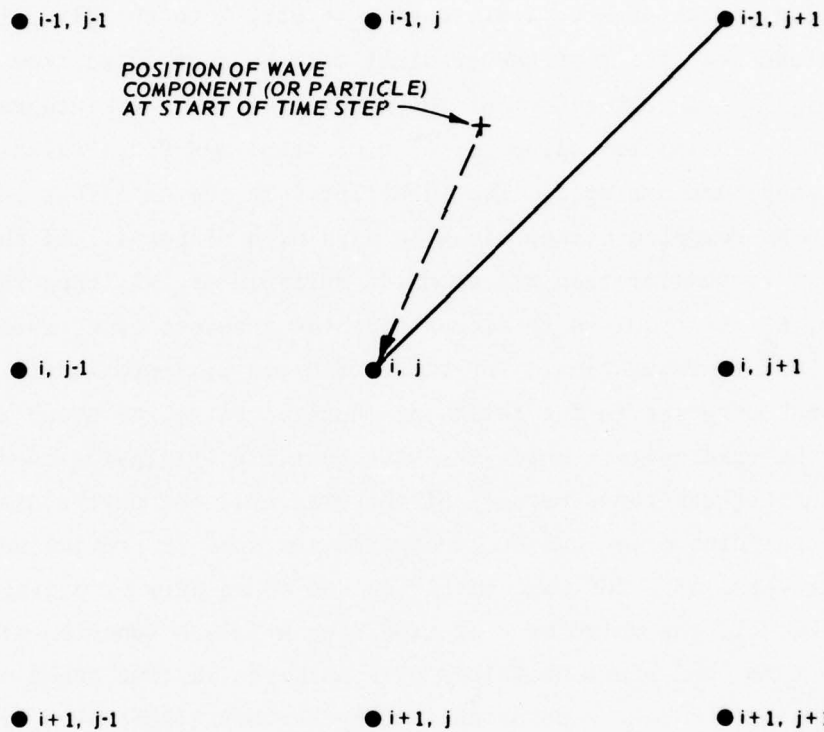
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Figure 2. Example of a particle position not along a diagonal or an axis at the start of a time step

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parameterized by Barnett; and ϕ represents the loss of energy due to interaction with the sea-bottom.

Unlike the representation of the source terms by Barnett, there is no limiting term present in Eq. 12. Field and laboratory studies (Sutherland, 1968; Barnett and Wilkerson, 1967; Barnett and Sutherland, 1968; Mitsuyasu, 1969; and Hasselmann et al., 1973) document a tendency for wave energy to "overshoot" its equilibrium value by a substantial amount before it stabilizes into a small oscillation about its equilibrium value, rather than a tendency to taper off into a slow approach to equilibrium from the low side. On the other hand, Shule et al., (1971) offers evidence that the "overshoot effect" may be an artifact created by the difference between the observed one-dimensional wave spectra and actual two-dimensional wave number spectra. In any case, there is no real agreement as to the nature and form of the limiting function as a spectral component approaches its equilibrium value. Figure 3 contrasts the results expected from models with a limiting term, a simple cutoff at equilibrium, and an "overshoot effect". For the purposes of the model presented here, the simple cutoff is adopted.

Eq. 12 can be expanded in a Taylor series so that the energy of a given particle at the end of a time step, Δt, is related to the energy at the start of the time step by

$$F_{ijk\ell}^{n+1} = F_o + \sum_{q=1}^{r} \frac{(\beta - \tau - \phi)^{q-1}}{q!} \left[\alpha + \Gamma + (\beta - \tau - \phi) F_o \right] \Delta t^q , \qquad (13)$$

where F_o is given by the right hand side of Eq. 11 without the source terms. A simple cutoff to the growth predicted by Eq. 13 is given by the constraint on the energy density in a one-dimensional spectrum as proposed by Phillips (1958) on dimensional grounds

$$E_{max} (f) = (2\pi)^{-4} a g^2 f^{-5} , \qquad (14)$$

where E_{max} is the maximum attainable energy density for a frequency f, g is gravity, and a is taken to be a universal constant. In order to
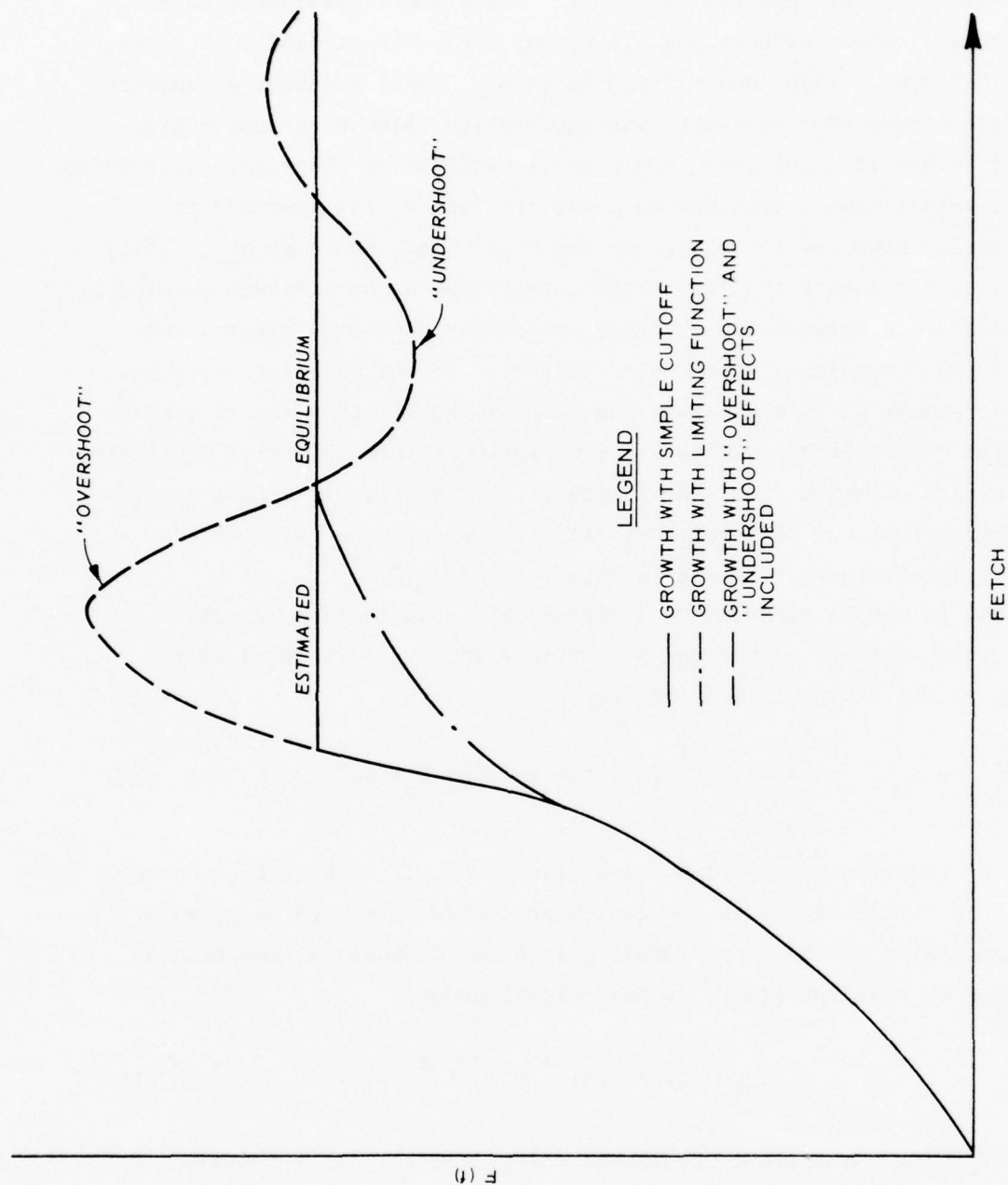
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Figure 3. Hypothetical growth with increasing fetch of a wave component at a given frequency for a simple cutoff, a limiting function or with "overshoot" and "undershoot" effects included

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apply Eq. 14 to the problem of wave generation, the two-dimensional spectrum at each grid point is integrated to obtain the one-dimensional spectrum.

If $E_{ijk} > E_{max}(f)$ then the energy density of the individual wave particle is normalized as follows:

$$F_{ijk\ell} = F_{ijk\ell} \cdot \frac{E_{max}}{E_{ijk}} \quad . \tag{15}$$

Up to this point, in spite of the differences in limiting functions, the model described in this paper gives results that are very similar to those of Barnett (1968). As shown in Figure 1, this does not provide a close fit to observed growth rates of wave height with fetch.

It appears that much of the discrepancy originates in the assumption that "a" in Eq. 14 is a constant. Studies by Mitsuyasu (1968b) and Hasselmann et al., (1973) have offered considerable evidence that the magnitude of "a" varies with nondimensional fetch, defined by Kitaigorodskii (1962) as

$$\bar{D} = \frac{gD}{u_*^2} \quad , \tag{16}$$

where D is the fetch and u_* is the friction velocity of the wind. Figure 4 shows that this variation is quite systematic and certainly not negligible over a wide range of non-dimensional fetches. The value of "a" used in both the FNWC and Barnett model is 0.0081 which as seen in Figure 4 is representative of relatively long fetches such as might be typical of open ocean conditions.

Hasselmann et al., (1973) suggests that the variation in Phillips' "equilibrium constant" is an effect of the adjustment of the spectrum to a level at which the energy input by the wind can be largely removed by the nonlinear energy transfer to shorter waves. The behavior of "a" is thus linked to the development of the spectrum along a fetch. As shown in Figure 1, there also is a relatively well-defined variation in nondimensional wave height with fetch, where the nondimensional wave height is defined as
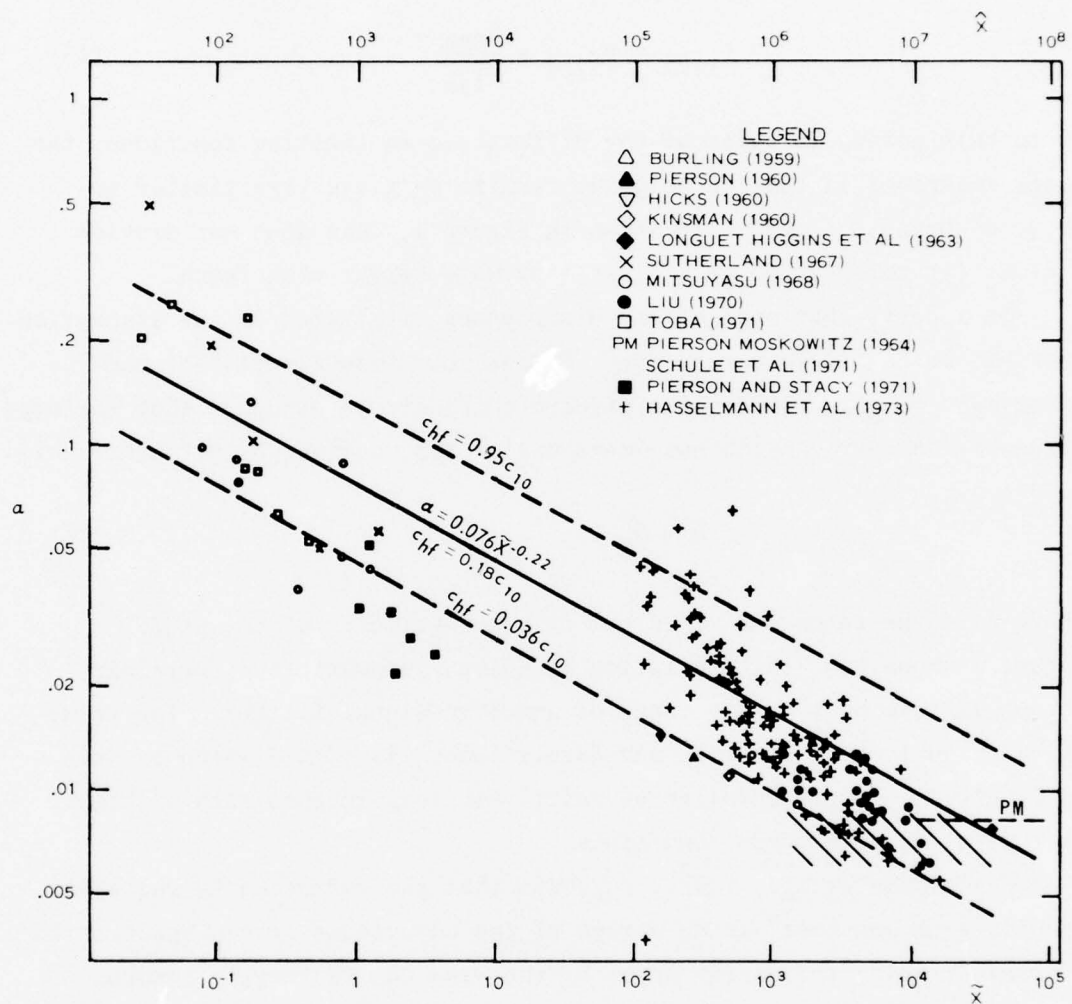
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Figure 4. Variation in Phillips constant with nondimensional fetch (after: Hasselmann et al., 1973)

$$\overline{H} = \frac{g\sqrt{E_o}}{u_*^2} \quad , \tag{17}$$

with E_o being the total energy within the spectrum. By combining the relationships among nondimensional fetch, "a" and nondimensional wave height, it is possible to parameterize the variation in "a" in terms of nondimensional wave height

$$a = b_1 \; \overline{H}^{\, b_2} \quad , \tag{18}$$

where b_1 and b_2 are arbitrary coefficients to be determined from empirical evidence. Two major difficulties are avoided by this reparameterization. First, in complex geometries, the definition of fetch is not a simple procedure. This is particularly true for cases in which the wind direction is shifting significantly during the time that the waves are generated. The use of parameterization for "a" based on local wave height and local wind speed permits a relatively simple evaluation of "a" even under circumstances such as these. Second, although often the largest (and consequently most important in terms of structure design) waves are fetch limited, there are many cases in which duration rather than fetch is the limiting factor and other cases in which the low frequency portion of the spectrum is fetch-limited while the high frequency portion is still duration-limited. In using nondimensional wave height to parameterize "a," any unrealistic assumptions regarding an "effective fetch" for such cases can be avoided.

The above discussion indicates that variations in "a" can be related in a more general sense and with less ambiguity to nondimensional wave height rather than to nondimensional fetch. However, since all of the previous studies of systematic variations in "a" have compared the values of "a" to nondimensional fetch, there is no a priori evidence that a similar relationship exists between "a" and nondimensional wave height, other than in the case where both are known functions of nondimensional fetch.

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In an attempt to determine whether or not nondimensional wave height provides a suitable scaling parameter for "a" several sources of published wave data were reviewed. Data from air-sea interaction experiments in the open ocean by DeLeonibus and Simpson (1972) and DeLeonibus et al., (1974) provided an ideal set of data with which to test this hypothesis. The study by DeLeonibus and Simpson (1972) documents a case study of duration-limited wave spectra at the Argus Island tower (Figure 5). The study by DeLeonibus et al., (1974) documents several sets of observations at the Argus Island tower over a variety of weather conditions. In neither study should the spectra be fetch-limited; and in both, the estimates of "a" are provided directly, as are direct measurements of wind speed at several levels.

Published data by Liu (1971) provided another source of information which could be used to derive simultaneous values of "a" and nondimensional wave height. As a final source of data, wave spectra provided by the Canadian Department of the Environment were processed to obtain estimates of "a". These final two sources of wave information are taken from the Great Lakes and each sample was screened to insure that the waves were not fetch-limited at the time the observation was made. The wind data from the Liu study were taken from wind velocities recorded at a tower located in Lake Michigan, near Muskegon, Michigan. The friction velocity was estimated from this wind measurement, under the assumption of neutral stability, using a technique adapted from Cardone (1969). The wind data for the Canadian wave spectra came from measurements made during the International Field Year on the Great Lakes (IFYGL) program. These were converted to estimates of friction velocity in a manner similar to the Liu data. Figure 6 presents some of the normalized Canadian wave spectra, indicating their tendency to reach a constant value in the equilibrium range. The value of "a" was estimated over the range of frequencies $0.25 \leq f < 0.3$.

Figure 7 shows calculated values of "a" from the Canadian wave spectra and values of "a" from published data plotted against $\tilde{\varepsilon}$, where $\tilde{\varepsilon} = \overline{H}^2$. There is strong indication of a systematic variation in the value
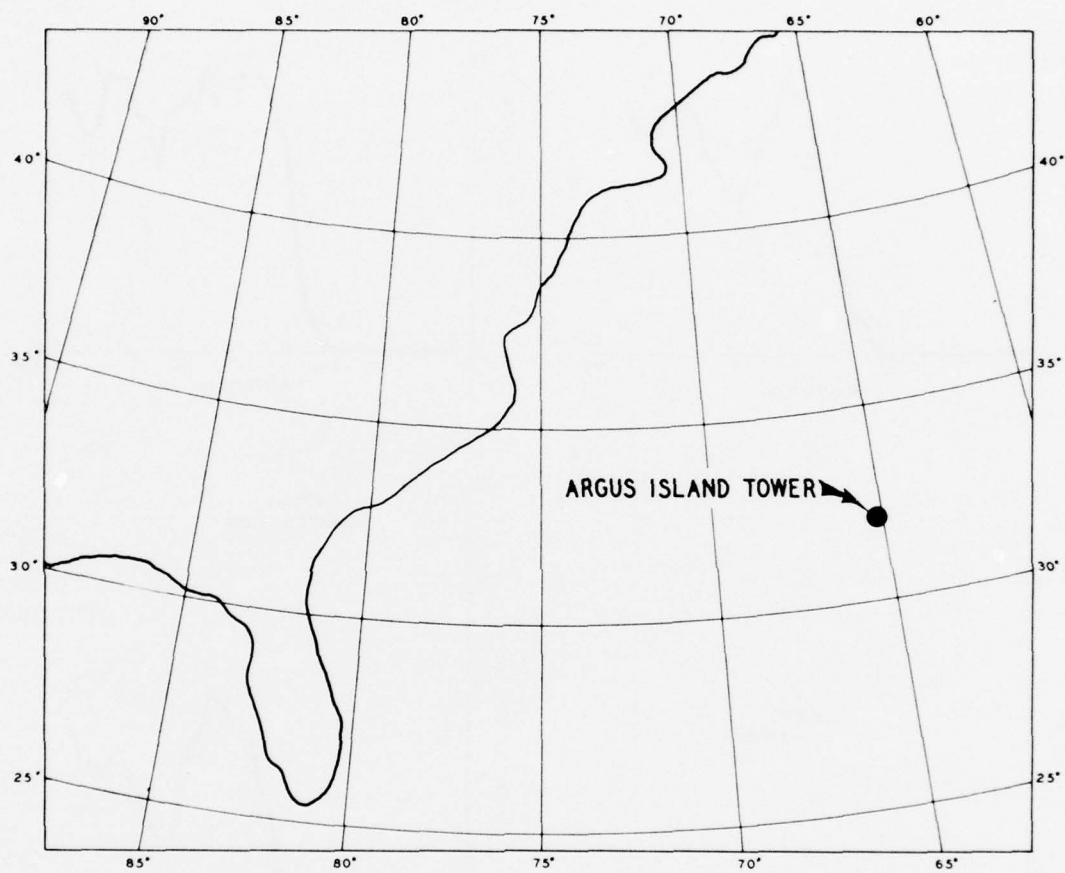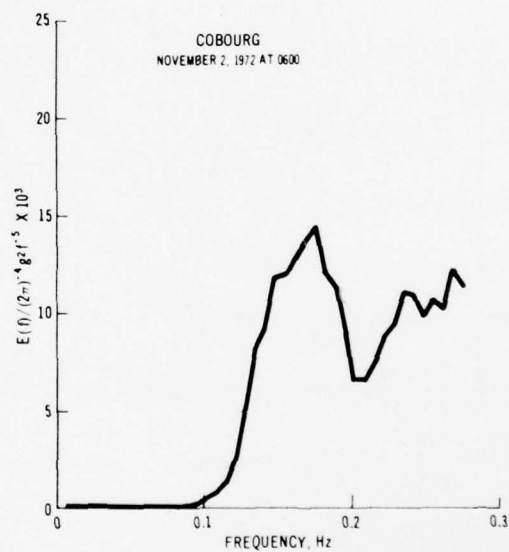
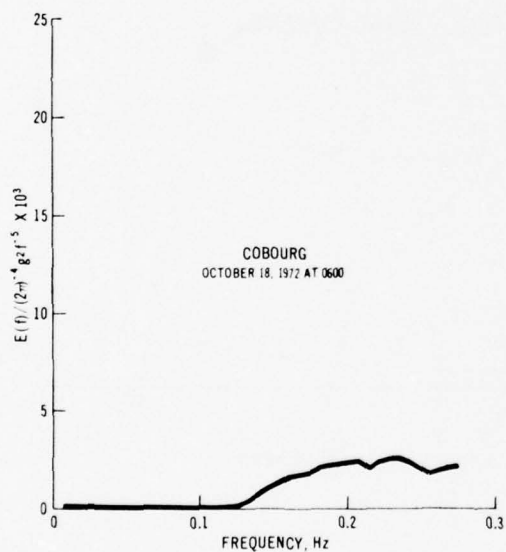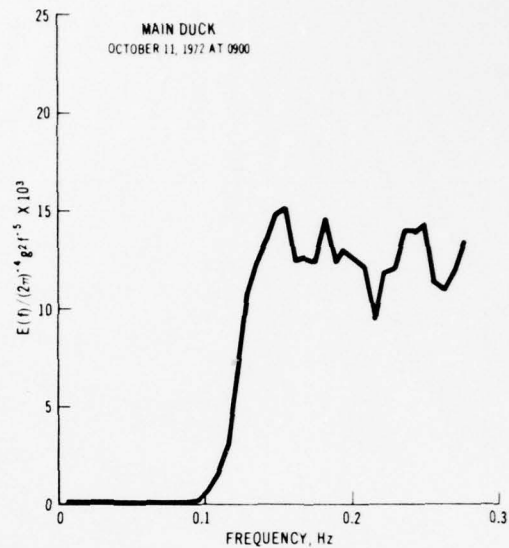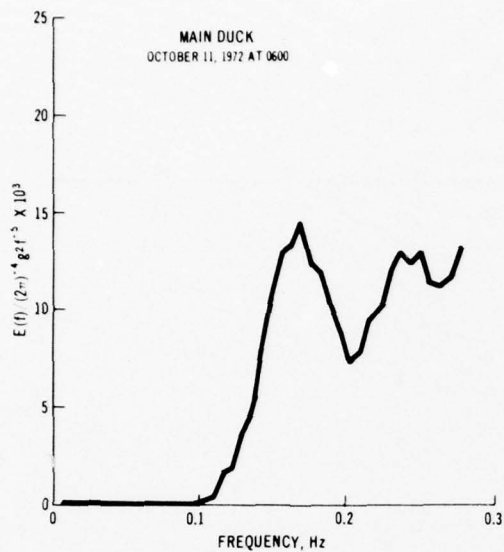Figure 5. Location of Argus Island tower

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Figure 6. Normalized spectra from unsmoothed FFT estimates at Main Duck and Cobourg wave gages in Lake Ontario

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of "a" related to different nondimensional wave heights. The constants
in Eq. 18 which give the best fit to these data are b_1 = .037 and
b_2 = -0.46. The relationship between nondimensional wave height and
nondimensional fetch as estimated by Hasselmann et al., (1973)

$$\bar{H} = 1.27 \times 10^{-2} \bar{D}^{0.50} \tag{19}$$

can be combined with Eq. 18 and these values of b_1 and b_2 to give

$$a = 0.27 \bar{D}^{-0.23} . \tag{20}$$

This relationship compares reasonably well with the formulas derived by
Hasselmann et al. (1973)

$$a = 0.31 \bar{D}^{-0.22} ; \tag{21}$$

Liu (1971)

$$a = 0.40 \bar{D}^{-0.25} ; \tag{22}$$

and Mitsuyasu (1973)

$$a = 0.59 \bar{D}^{-0.31} . \tag{23}$$

Whether or not the nondimensional wave height is a more general
scaling parameter than nondimensional fetch cannot be answered on the
basis of the small sample shown in Figure 7. However, there is good
indication that nondimensional wave height can provide a reasonable
first approximation to the scaling of "a" in a numerical model. Hence,
the general form of the total energy at a grid point at the end of a
time step can be expressed as

$$E_0 = \int_0^{f_c} \int_0^{2\pi} F(f,\theta) \, df d\theta + \int_{f_c}^{f_h} a(2\pi)^{-4} g^2 f^{-5} \, df \tag{24}$$

where f_c represents the frequency at which the spectrum reaches the
equilibrium range and f_h is simply a high frequency cutoff, somewhere
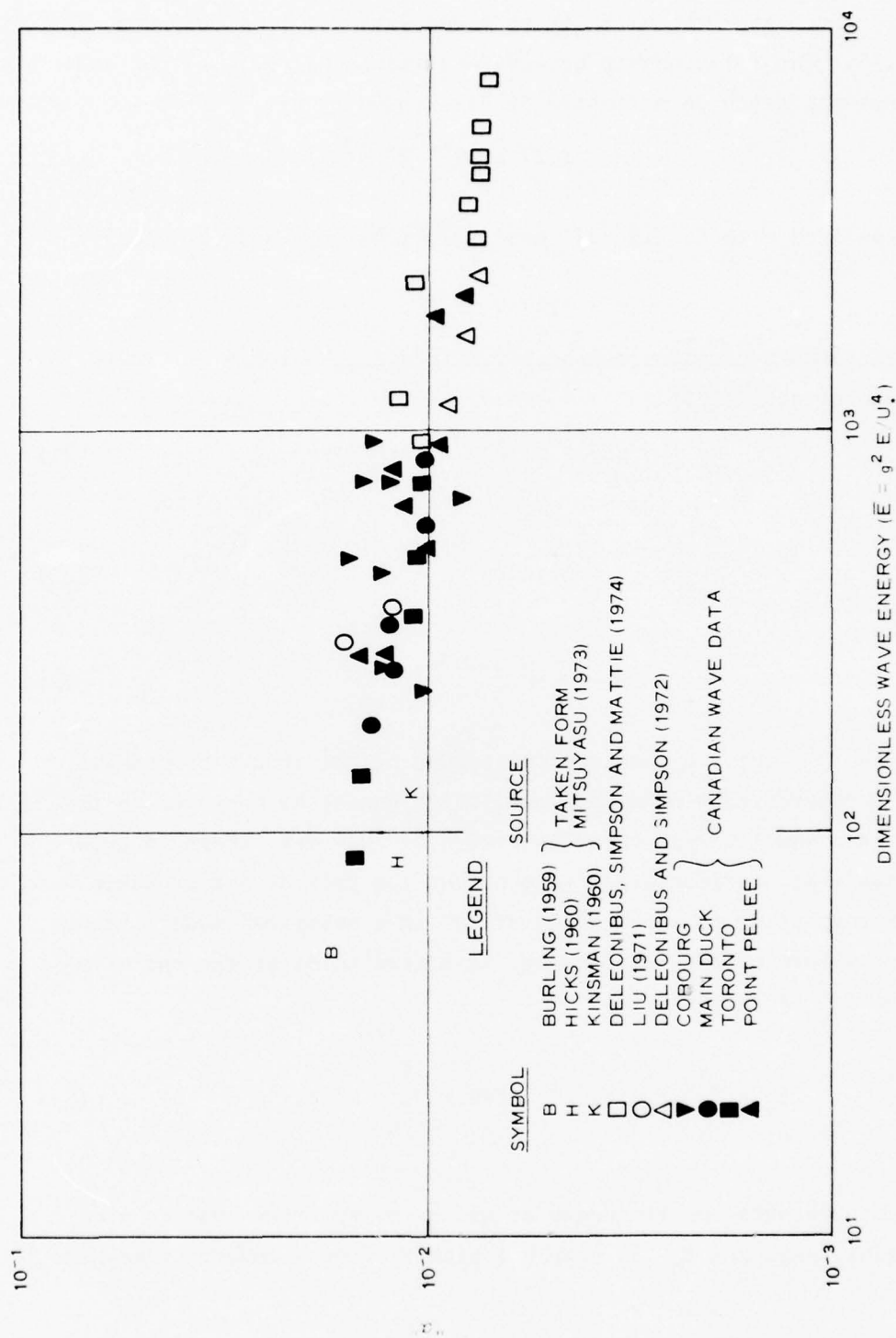
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Figure 7. Variation in Phillips constant with nondimensional wave energy

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above the Kitaigorodskii range, as defined by Pierson and Stacy (1973).
Substituting the values b_1 and b_2 into Eq. 18 gives

$$a = 0.37 \, \overline{H}^{-0.46} \qquad (25)$$

Eqs. 24 and 25 can be solved using an iterative technique to provide
estimates of E_o , a and f_c .

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PART III: TEST OF NUMERICAL MODEL

The numerical procedure described in the previous section was
programmed on a CDC 7600 computer. The set up of the program requires
a grid of bathymetric data, a single value denoting grid spacing, desired
frequencies for spectral calculations, and wind speeds and directions.
Since the model output is to be compared to results of studies in deep-
water bordered by a straight shoreline, a deep rectangular lake was chosen
for testing (Figure 8). The grid point spacing is 10 miles and the
dimension of the lake is 220 miles by 120 miles. A 15-minute time step
was chosen to meet the constraint that no energy propagated over 10 miles
during a single time step. The depth was set to a constant 1000 meters.

Winds of 10-, 20-, and 30-mps, blowing parallel to the short sides
of the lake, were input into the computer program. Figure 9 gives the
results of computations for points along the centerline of the lake.
The agreement with the field data is good across the entire range of
nondimensional fetches.

The growth of wave heights with time for different wind speeds can
be compared by graphing nondimensional wave height against nondimensional
time, defined as

$$\bar{t} = \frac{gt}{u_*} \tag{26}$$

Figure 10 shows the results of this comparison and indicates that the
growth rates for different wind speeds are quite similar. Also shown in
Figure 10 are nondimensional wave heights as a function of nondimensiona
time from the I-B model, Mitsuyasu and Rikiishi (1975), and Hasselmann
et al. (1976).

The data shown in Figure 10 for the numerical model presented in
this paper are taken from early portions of the generation sequences in
the rectangular lake, where the effects of fetch limitation are not yet
significant. The results of the I-B model are taken from published
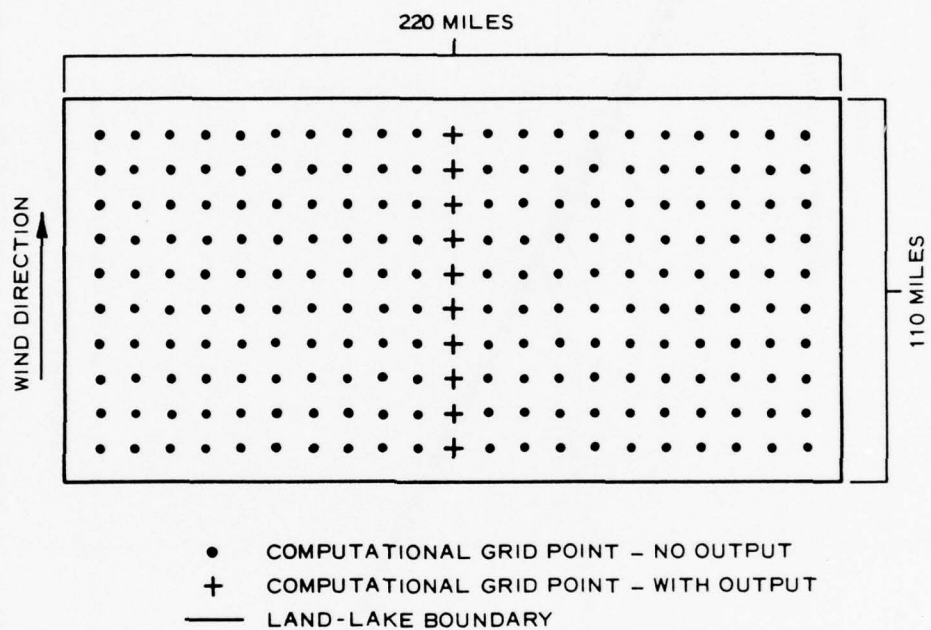
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Figure 8. Grid set-up for rectangular lake test of modified-Barnett source terms in the numerical model presented in Part II
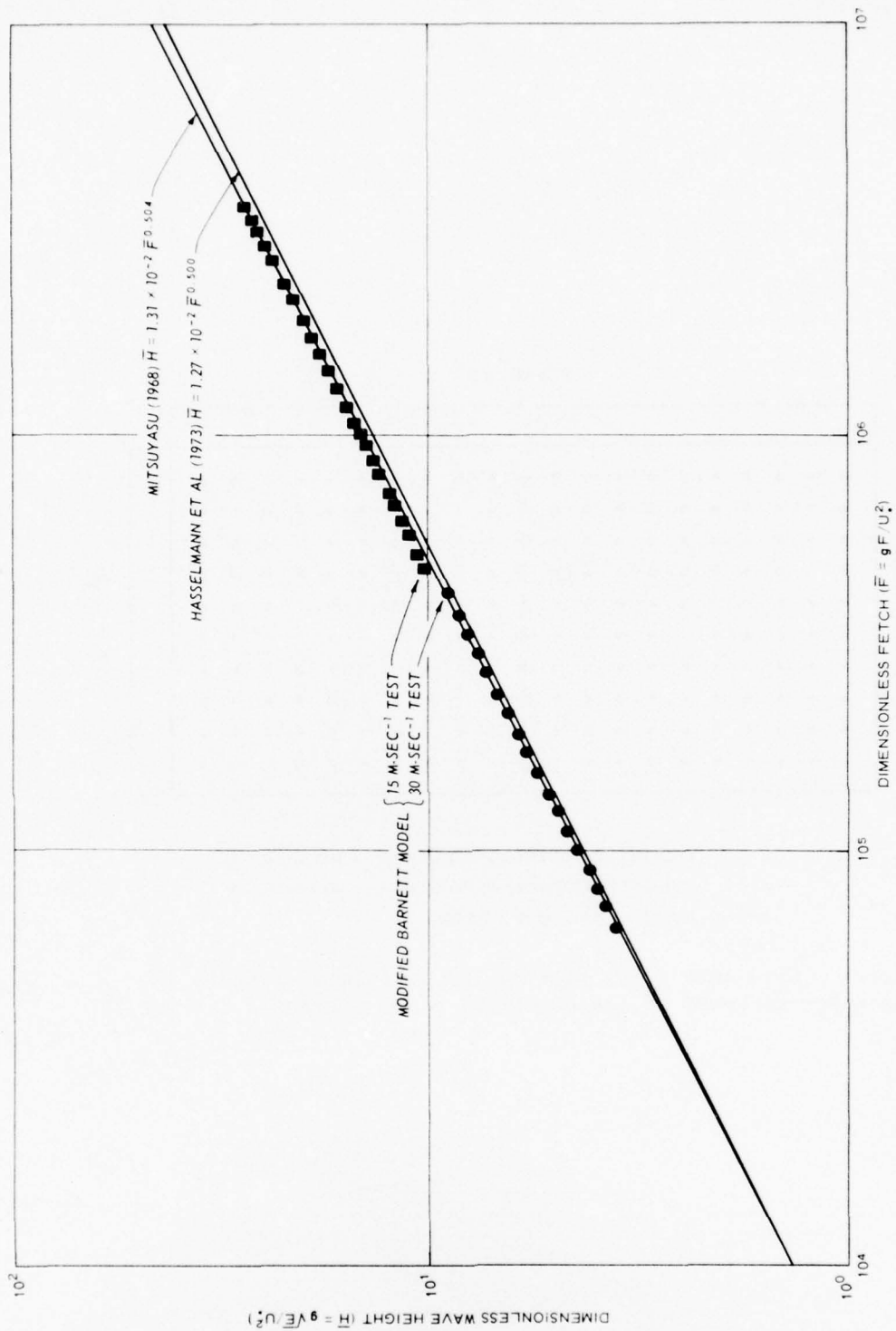
Figure 9. Comparison of nondimensional wave heights from numerical model presented in Part II to relationships between nondimensional wave height and nondimensional fetch derived from observations by Mitsuyasu (1968b) and Hasselmann et al. (1973)

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curves by Inoue (1967). The results of Mitsuyasu and Rikiishi (1975) are based on extrapolations from their laboratory results,

$$\overline{H} = 1.17 \times 10^{-4} \, \overline{t}^{\,1.21} \quad . \tag{27}$$

As seen in Figure 10, Equation 27 predicts extremely large wave heights at moderate nondimensional times (1 to 10 hours with wind speeds from 10 to 20 m/sec). For example, using the relationship given in Eq. 27, one obtains an estimate of 6.7 meters for a 10 m/sec windspeed and a duration of 1 hour. This is much too high for rates of growth observed at scales larger than those encountered in the laboratory study. The results shown in Figure 10 for Hasselmann et al. (1976) are based on the solution to a parametric radiative transfer equation. If a substitution of $u_* = 0.0316 \ u$ is made into their parametric equation one obtains

$$\overline{H} = 1.90 \times 10^{-3} \, \overline{t}^{\,0.714} \quad , \tag{28}$$

which deviates remarkably from the laboratory growth rates, but does come fairly close to the growth rates of the numerical models.

In Figure 10 the lines from the I-B model and the model proposed in this paper are disjoint. This indicates that the wave growths with time do not follow a similarity law in either of these models, in spite of the fact that the model presented in this paper does follow a similarity law for wave growth with fetch. It should be emphasized here that there is a fundamental difference between the uses of stochastic analyses in the fetch-limited and time-limited cases. In the former situation the sequence of water surface elevations at a given location can be regarded as a stationary random function. In the later situation, this is not true since the random process is non-stationary. An argument can be made that the process will still follow a similarity law in its development if one assumes that the process can be broken into sufficiently small increments such that the process is stationary over each increment. However, this hypothesis needs to be proven on the basis of considerable laboratory and prototype
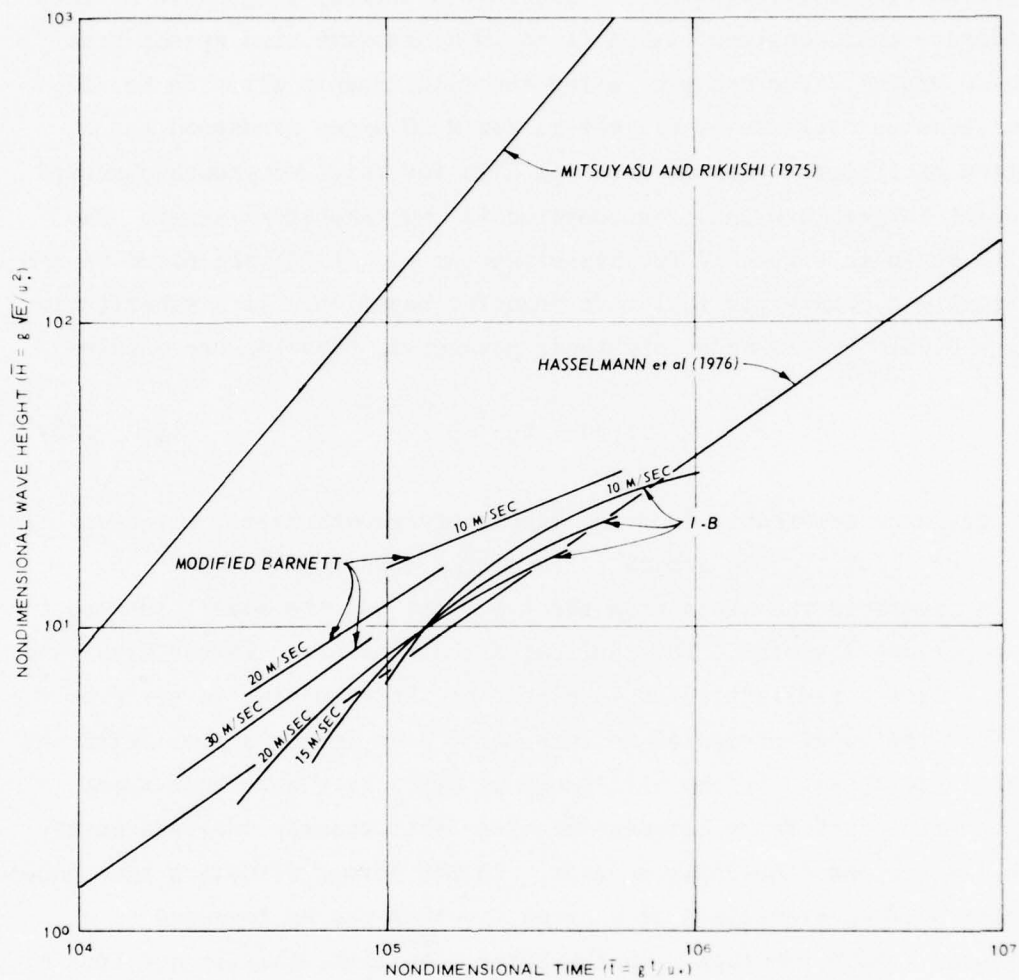
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Figure 10. Comparison of relationships between nondimensional wave height and nondimensional time from numerical models and laboratory and prototype-scale observations

data. To date there has been no evidence presented which confirms this hypothesis. In fact, just the opposite appears to be the case from the results of Mitsuyasu and Rikiishi (1975) who conclude that, even within the narrow range of nondimensional times in their experiment, the similarity of the one-dimensional spectrum was not satisfied for duration-limited waves.

As seen in Figure 10 the recent parametric model developed by Hasselmann et al. (1976) does follow a similarity law; however, their model is not based on an extensive calibration with duration-limited waves, but rather on the assumption that spectral shape is constant. This assumption is not supported by the data of Mitsuyasu and Rikiishi (1975). Figure 11 gives the spectra of Mitsuyasu and Rikiishi for fetch-limited and duration-limited conditions normalized in the forms

$$\frac{E(f) \ f_m}{E} = \overline{E} \ (\frac{f}{f_m}) \ , \tag{29}$$

and

$$\frac{E(f) \ f^{-5}}{g} = \tilde{E} \ (\frac{f}{f_m}) \ , \tag{30}$$

where $\overline{E}$ and $\tilde{E}$ are functions of relative frequency. The close similarity among the fetch-limited spectra is not duplicated among the duration-limited spectra. Not only are the duration-limited spectra quite dis-similar among themselves, but they also differ substantially from the form of the fetch-limited spectra. The duration-limited spectra are consistently less peaked than the fetch-limited spectra. This same tendency for a decrease in the peakedness of the spectrum is supported by the relationship, based on data from several field studies, between the Hasselmann et al. (1976) peak enhancement parameter and nondimensional peak frequency defined as

$$\overline{f}_m = \frac{u_* f_m}{g} \ , \tag{31}$$
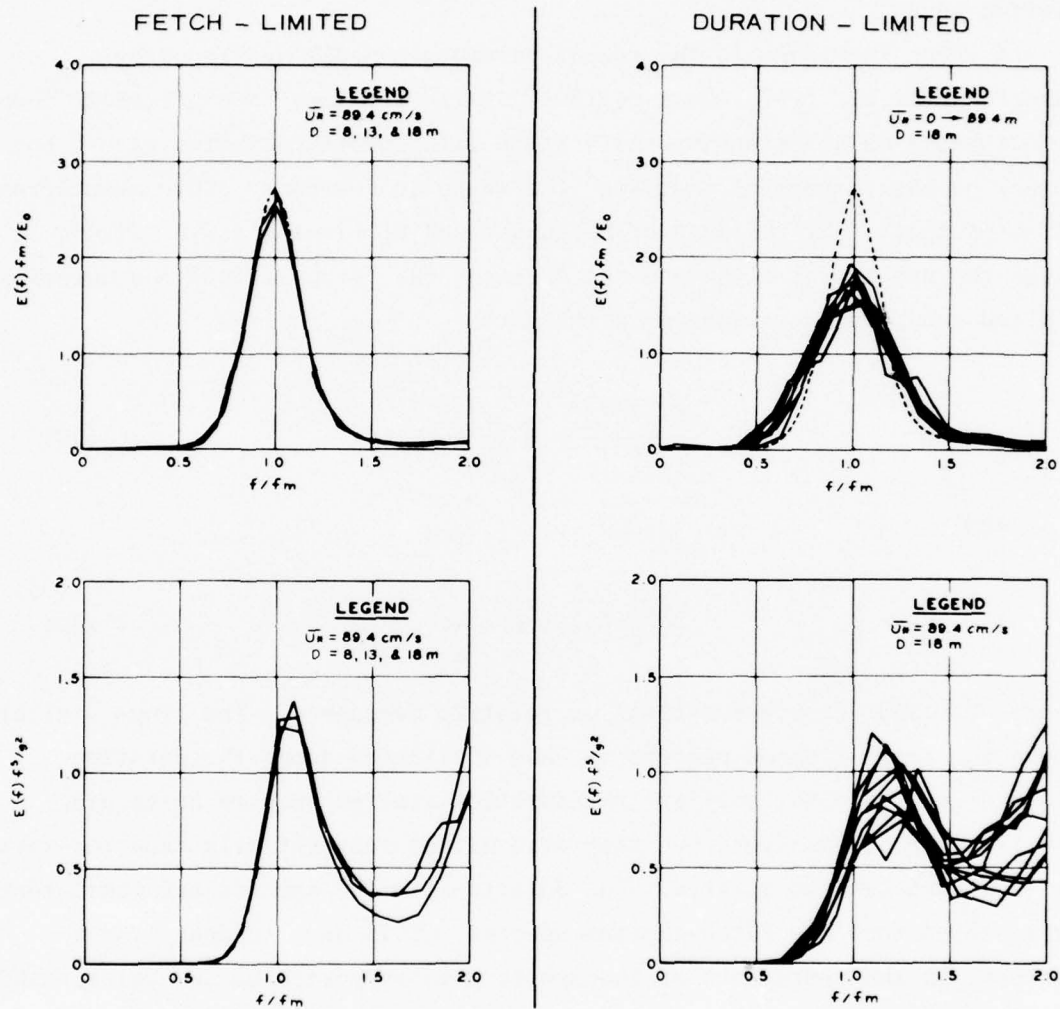
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Figure 11. Comparison of normalized spectra from Mitsuyasu and Rikiishi (1975) for fetch limited and duration limited conditions

where f_m is the frequency of the spectral peak. The Hasselmann et al. (1976) results indicate a significant decrease in spectral peakedness as $\overline{f}_m$ decreases. Since nondimensional frequency is an inverse function of nondimensional time, this again indicates a broadening of the spectrum as nondimensional time increases.

It is difficult to reconcile these differences among the models and laboratory evidence. It is possible that, if the duration-limited spectra are dissimilar, the range of nondimensional times used in calibrating a similarity growth law will affect the exponents used in the relationship. If the spectrum truly broadens with increased non-dimensional time, the related reduction in magnitude and displacement in frequency of the nonlinear wave-wave interaction source term might explain the tendency to grow only to a "fully-developed" spectral form as hypothesized by Pierson and Moskowitz (1964). The possible differences between several available open-ocean models, all of which incorporate some concept of a fully-developed spectrum, and the growth rates of Eqs. 27 and 28 are shown in Figure 12. It would appear that the Mitsuyasu and Rikiishi (1975) relationship fits the growth rate in the early stages of wave generation, whereas the Hasselmann et al. (1976) relation-ship fits the growth rate in the intermediate stages of wave generation. Neither relationship agrees with the growth rates of the models for late stages of wave generation.

As a final check of the consistency of the model estimates and estimates from Eqs. 27 and 28 with some prototypescale data, the observations from the duration-limited study by DeLeonibus and Simpson (1972) were compared to predicted wave heights (Figure 13). For the period reported in the DeLeonibus and Simpson study, the windspeed at 10 meters was approximately constant at 10 to 11 $m\text{-sec}^{-1}$ for 15 hours. During that time, the significant wave height grew from 1.0 meter to 2.7 meters. Although local variations in the air-sea temperature difference created variations in the coefficient of drag, and hence, in the estimated friction velocity, the average Richardson Number may have been relatively constant when averaged over the entire region of wave generation. Consequently,

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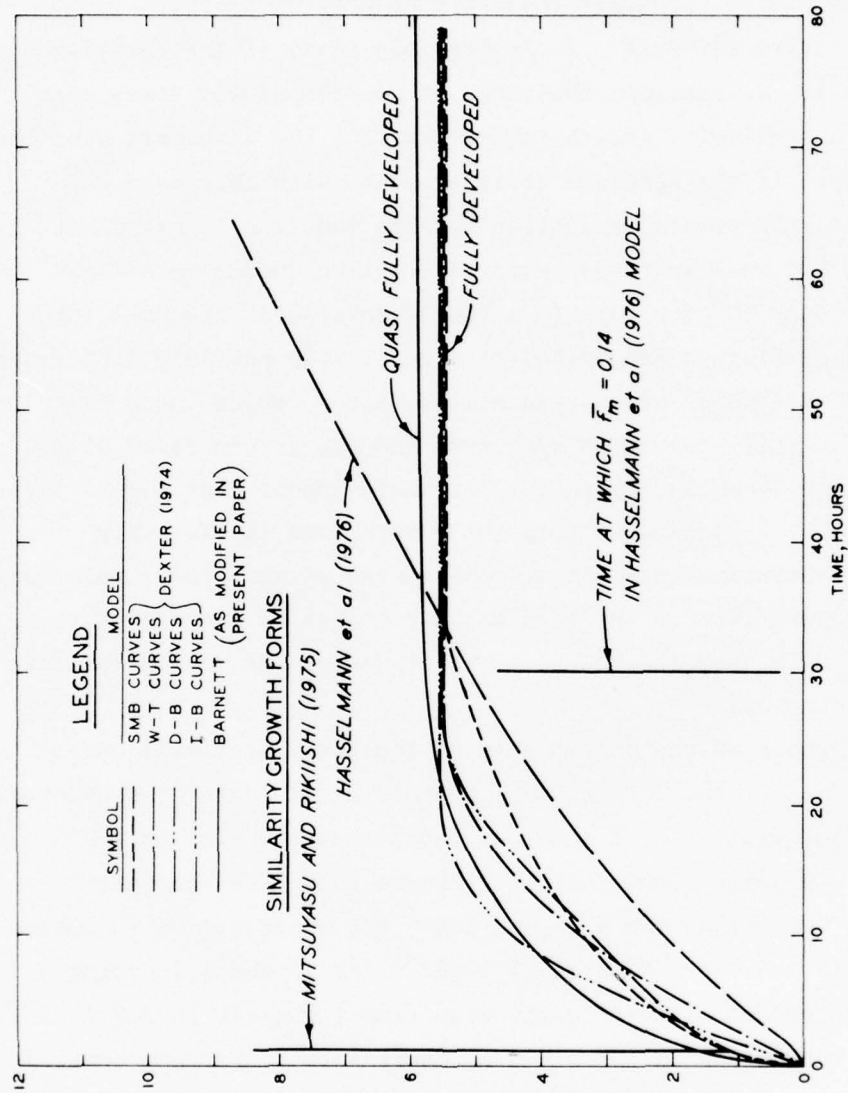
Figure 12. Growth rates of significant wave height with time from several numerical models tested by Dexter, the model presented in Part II with the modified-Barnett source terms and laboratory- and prototype-scale empirical formulae
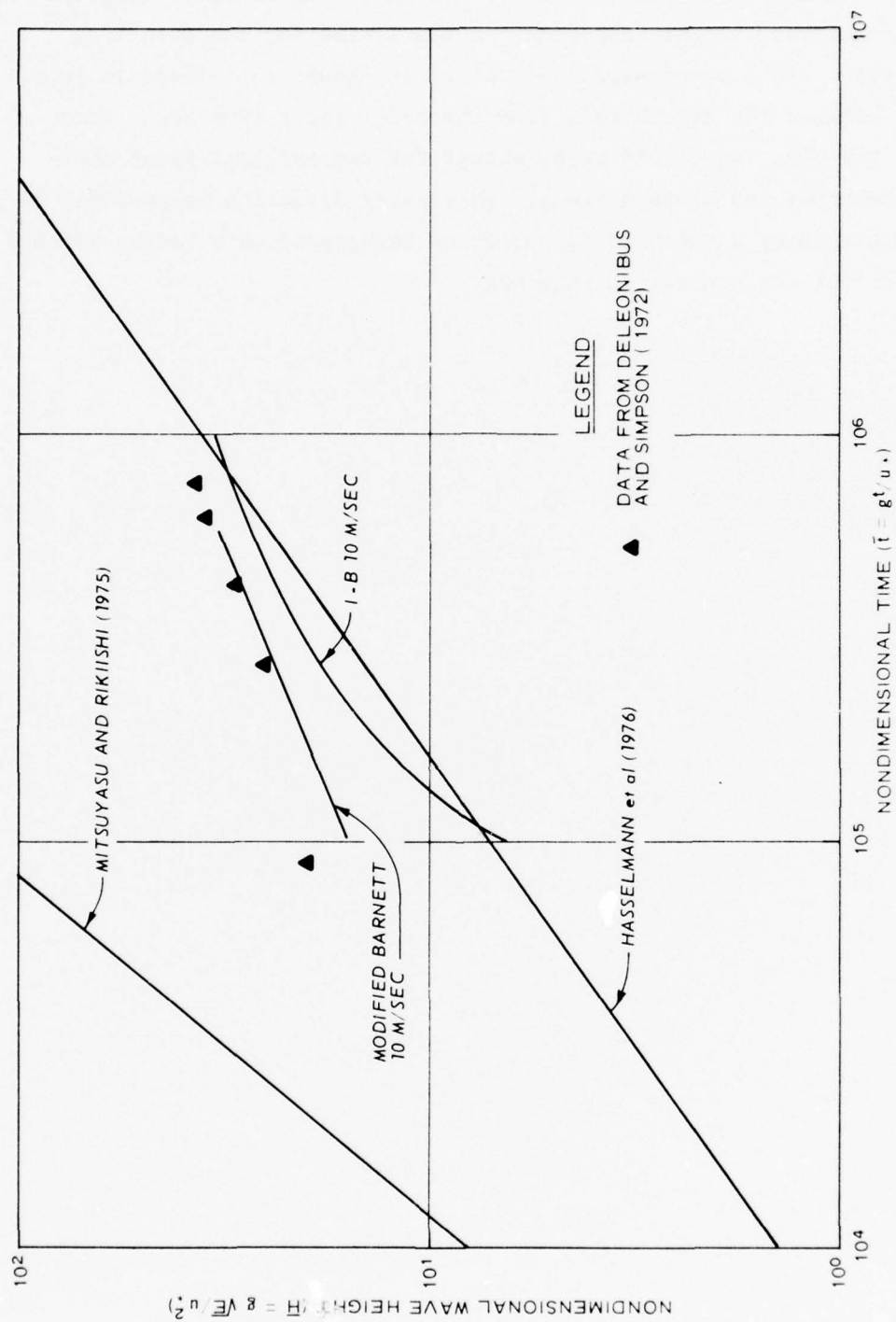
Figure 13. Comparison of nondimensional wave heights from (a) Mitsuyasu and Rikiishi (1975), (b) Hasselmann et al. (1973), the FNWC model, and the modified-Barnett model to observations by DeLeonibus and Simpson (1972)

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a value of 0.41 m-sec^{-1} was taken to be representative of the friction
velocity for the 15-hour period. This value was used in estimating the
nondimensional wave heights and nondimensional time for the data from
the DeLeonibus and Simpson study as plotted in Figure 13. There is good
agreement between the growth rate from the model for a 10 m-sec^{-1} wind-
speed and the observed growth rate, except for the earliest point shown
in the DeLeonibus and Simpson study. This early deviation is possibly due
to contamination by a one-half to one-meter background wave height present
at the start of the generation sequence.

PART IV: DISCUSSION

In attempting to perform the calculations involved in the rectangular-lake test, some computational difficulties emerged. As pointed out previously, the assumption of a homogeneous gradient is strictly valid for a linear energy gradient over a distance $2c_g \Delta t$. For constant values of $\alpha, \beta, \Gamma, \tau$ and ϕ , the solution along a wave particle path with no limiting term can be represented as

$$F^2 = \frac{(\alpha + \Gamma + 1) \exp (\Delta t') - \alpha + \Gamma}{\delta'} , \qquad (32)$$

where F_1 and F_2 are the energy densities at the beginning and end of the time step, respectively, $\Delta t'$ is the time required to travel from one grid point to the next, and δ' is given by

$$\delta' = \beta - \tau - \phi . \qquad (33)$$

It is clear from Eq. 32 that when the spectrum is approaching its fetch-limited form, the gradient can become highly nonlinear. The effect of this nonlinearity is to underestimate the gradient at a grid point. Consequently, the energy is allowed to exceed the actual fetch-limited amount. Next to a solid boundary, this introduces a small excess of energy which then can propagate across the entire fetch, creating a slow energy source that eventually can dominate the entire estimate of growth across a fetch. This problem can be handled if the energy at a grid point is always limited to that estimated in Eq. 32. This only influences the solution when the spectrum approaches its fetch-limited form and does not require much additional time for computation.

Another problem encountered during the program testing was the range of convergence for Eq. 13. Practical considerations limit the number of terms in this equation; thus a reasonable criterion of convergence is given by

$$|\delta'|\Delta t < 1 \quad , \tag{34}$$

where Δt is the size of the time step in the model. τ is usually very small in the frequencies undergoing rapid growth and can be neglected in considerations of convergence. As indicated by Inoue (1967) and Hasselmann et al., (1973) the β-term can be written in the form

$$\beta = n_1 \left(\frac{u_*}{c}\right)^{n_2} \cdot 10^{-3} f \quad , \tag{35}$$

where c is the phase velocity and n_1 and n_2 are constants. For the range of interest in wave generation, β can be approximated for a self-similar spectrum as (Hasselmann et al., 1973)

$$\beta \simeq 6\left(\frac{u_*}{c}\right)^{1.33} \cdot 10^{-3} \cdot f \quad , \tag{36}$$

based on scaling laws of the frequency of the spectral peak and the development of the spectrum with fetch. Even a conservative approximation limits the magnitude of β to at most

$$\beta \simeq 10^{-3} \cdot f \quad . \tag{37}$$

For neglegible bottom interaction, Eqs. 37 and 40 can be combined to give

$$\Delta t < 10^3/f \quad . \tag{38}$$

This criterion was met in the rectangular lake test since the time step was 900 seconds and the frequency range of interest went up only to 0.4 hz. Although the magnitude of ϕ was not investigated in this test of a deep lake, it is felt that convergence should not be a major problem in depths greater than 10 meters, for the range of wave frequencies typically important in such depths.

Another potential problem in the solution of the radiative transfer equation is the uncoupled, homogeneous form of the parameterized

wave-wave interaction terms. These terms are dependent on the total energy in the spectrum and the shape of the spectrum both of which change with position along a particle path. By evaluating these parameters at the mid-point of the path, the possible error is minimized. However, the time rate of change of the spectrum at the mid-point must still be considered as is required in Eq. 8. This can be accomplished by calculating a time-averaged value for the nonlinear terms over a time step with an explicit formulation of the type

$$\Gamma^{n+1/2} = \Gamma^n + \frac{\Gamma^n - \Gamma^{n-1}}{2} \quad , \quad \text{and}$$

$$\tau^{n+1/2} = \tau^n + \frac{\tau^n - \tau^{n-1}}{2} \quad . \tag{39}$$

The values of $\Gamma^{n+1/2}$ and $\tau^{n+1/2}$ are those taken to be used in Eq. 13. This formulation is quite stable since the rate of change of the total energy (Γ and τ in Barnett's parameterization is proportional to the total energy cubed and squared, respectively) is much less than the rate of change of individual spectral components.

There is no direct limit on the size of the grid spacing other than that

$$c_{g_{max}} \cdot \Delta t < \Delta x \quad , \tag{40}$$

where $c_{g_{max}}$ is the group velocity of lowest frequency included in the calculations. Anywhere beyond this limit, it is essentially a matter of resolution which must be determined. In areas of complex geometry, often the dominant factor to consider is the largest size of the grid cells which is still adequate to represent the major features of the waterland boundaries. However, the core storage and run time required by the program must also be considered when determining this grid size. For the model used in the rectangular lake calculations, 20 frequencies and 12 directions of propagation were used to characterize the wave generation. Since there were 210 grid points in this lake, the two basic working matrices in the model required 58,400 positions of core storage apiece. The run time on

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the CDC 7600 computer was approximately 1.2 seconds per hour of prototype calculations.

This paper thus far has been concentrated on problems encountered in setting up the numerical model and a solution has been presented which appears to give an adequate representation of wave growth. A different but equally relevant topic is the correctness of the source terms used in the model. Hasselmann (1968) discusses at great length the theoretical and empirical support for various source terms; however, since it is impossible, at present, to measure these terms separately, it is difficult to estimate their relative magnitudes. Even with this lack of precise information on wave generation, there are some known deficiencies in the source terms used in the model which should be pointed out at this time. The Barnett form for β was taken from Snyder and Cox's (1965) work in which the β-term was used to explain all of the nonlinear growth of the wave spectrum. The term is too large since it includes effects of the wave-wave interaction terms in its empirical evaluation. However, the magnitude of βF becomes equal to Γ only as F approaches its equilibrium value; therefore, the effect of the overestimate of β is not too severe, particularly since the value of E_{max} is rapidly attained and the growth is stopped.

The effect of the system of equations given by Eqs. 11, 24 and 25 is to produce a numerical model which is very similar in behavior to the parametric model of Hasselmann et al. (1976). There is a rapid adjustment to the saturation energy for frequencies higher than the frequency of the spectral peak; and there is a slow migration of the spectral peak toward lower frequencies. It is reassuring to note that the rate of change of the spectral peak appears to be relatively correct since it fits the same empirical relationships (Figure 1) as the Hasselmann et al. model, even though the model was not explicitly derived to fit these data. This implies a fairly good approximation to the rate of energy and frequency distribution of energy input into the spectrum for the model presented in this paper. Consequently, for relatively simple wave

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generation situations, the model presented here will give very similar results to the Hasselmann et al. model.

Unlike the Hasselmann et al. model, there is no assumption in the model presented in this paper regarding the instantaneous equilibrium of the wave components with frequencies higher than the frequency of the spectral maximum. In areas of complex shoreline geometry and rapidly varying wind directions and velocities, transient terms can develop which create significant deviations from an expected local equilibrium. In areas such as these, the solution of the complete radiative transfer equation is clearly preferred to the use of a one-parameter model.

A second problem in the Barnett parameterization of the source terms is the assumption of a "fully developed" Neumann spectrum (proportional to f^{-6} at high frequencies) for the spectral shape when the nonlinear transfer of energy due to wave-wave interactions were calculated. Although the general rate of transfer is not too affected by this, the peakedness of the distribution of transfer rates between the parameterized source term and the complete integral form is not duplicated well. However the transfer rates are qualitatively realistic and should approach those of the full integral form for most spectra.

Since the I-B model does not contain any source terms due to nonlinear wave-wave interactions, there is expected to be a marked difference in the two models; however, to some extent, the empirical evaluation of the β-term in the I-B model compensates for this difference, at least in relatively simple wave generation situations. However, the Barnett model does not require an empirical limit to wave growth on the forward face of the spectrum. Its source terms permit reaching a "quasi-equilibrium" in this range of frequencies without any arbitrary limiting factors. On the other hand, the I-B source terms would permit wave growth at low frequencies well beyond the values of the Pierson-Moskowitz "fully-developed" spectrum, if wave growth were not constrained to stay at that level. This suggests that the I-B model does not represent the actual transfer mechanisms with its source terms, but rather, represents a time-fetch scaling relationship between zero energy and the fully-developed spectrum. This is not necessarily a problem in open-ocean applications,

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but certainly limits its usage in smaller areas with irregular shaped boundaries.

Although the bottom interaction term appears theoretically justifiable (Hasselmann and Collins, 1968) and has been tested somewhat in field studies (Barnett 1969; Collins and Wier, 1971), there has been no single study which has truly demonstrated its validity, at least in terms of absolute magnitude. The coefficient of drag implicit in this term is usually taken to be about 0.01; however, the Hasselmann et al, (1973) study found that the present mechanism did not explain all of the phenomena observed, particularly in regards to the lack of modulation by tidal currents.

Perhaps the least understood process in wave generation is that of wave breaking. In this study, the role of wave breaking is assumed to be that of establishing an equilibrium range in which f^{-5} distribution of energy density exists. The effect of different angular distributions of energy, the mechanism of the overshoot phenomena, and the reason for the variation in the equilibrium "constant" , "a" , with nondimensional fetch (or nondimensional wave height) have not been investigated. Hasselmann et al. (1973) suggests that the variation of "a" is an effect of the adjustment of the spectrum to the level at which the energy input from the wind in the central portion of the spectrum is balanced by the transport of energy by nonlinear wave-wave interactions to lower and higher frequencies. On the other hand, the theoretical and experimental work of Phillips and Banner (1974) has indicated that increased energy in waves of low frequencies tends to reduce the energy in the equilibrium range. Either phenomenon might qualitatively explain the observed variation in "a". The important point, relative to the purpose of the numerical model, is that the functional relationships in the variation of "a" are reproduced in the model regardless of whether wave-wave interaction or wave breaking is responsible for this variation.

A possible alternative for the numerical wave model would be to adopt a representation of wave breaking similar to that given by Hasselmann (1974)

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$$S_B = -\text{constant} \cdot f^2\, E(f) \quad , \tag{41}$$

where S_B is an energy sink due to white capping. Since the form of this equation contains E to the first power, S_B could be included in an expansion series by using an approximation to the angular distribution of energy over a time step, i.e.

$$\frac{\partial F}{\partial t} = \alpha + \Gamma + (\beta - \tau - \phi - \mu)\, F \quad , \tag{42}$$

where μ represents the energy loss along a particular propagation direction due to wave breaking.

In spite of all these uncertainties in the mechanics of wave generation and dissipation, open-ocean models for wave prediction and hindcasting have proven to be reasonably reliable, with possibly the major limiting factor that of obtaining accurate wind-fields over the water. This paper has extended the concepts applied in these open-ocean models to a more general form appropriate for complex shoreline geometries and large ranges in nondimensional fetch. Since, as seen in Figure 8, wave generation in the open-ocean almost always results in comparable magnitudes of nondimensional wave heights $\overline{H}$, the effect of variations in the Phillips coefficient "a" are not too apparent in open-ocean hindcasts. However, neglect of this variation in areas where fetches are limited can lead to serious mis-estimation of the wave energy in the spectrum.

PART V: CONCLUSIONS

Previous studies have concentrated their efforts on numerically modeling wave generation in the open ocean. This paper has attempted to generalize this approach to obtain a functional model in areas of complex shoreline geometries. The numerical technique presented provides a solution for any set of source terms which are expansible in a series with a small number of terms. The approximations involving homogeneous, linearized forms of the nonlinear source terms appear to provide an adequate representation for wave growth, without creating problems with the stability of the solution.

The nondimensional growth rates with nondimensional fetch estimated by this model are in good agreement with results from recent laboratory and field experiments; and the growth rates with time seem consistent with field data. A major revision in the Barnett source terms necessary to obtain this agreement is the inclusion of a dependence of the Phillips equilibrium parameter on nondimensional wave height. This modification leads to a model which for simple wind-wave fields gives growth rates and spectral shapes very similar to the recent parametric model of Hasselmann et al. (1976), for wave growth along a fetch, but differs markedly in the duration-limited growth characteristics. It would appear that considerably more field data will be needed to provide more conclusive results for duration-limited wave growth.

The I-B source terms performed very poorly in reproducing the JONSWAP growth rates with fetch. It would appear that the I-B model functions more as an approximate time-fetch scaling parameter than as an analogue to the wave-growth process. For this reason the modified Barnett source-terms are chosen for application of the numerical model in complex wave generation situations, even though there are some known deficiencies in the source terms as formulated by Barnett (1968). A first step toward removing these would be to parameterize the wave-wave interaction in a more generalized fashion and then to recalibrate the β-term with the growth due to the wave-wave interactions subtracted. Also, additional research, particularly regarding the bottom interaction

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and role of wave breaking, is needed to provide an improved representation in very shallow areas. However, the present forms of all these processes are probably adequate for a first estimate, at least up to the point that depth-induced wave breaking becomes significant. It should be emphasized that the present form of the model does not include the effects of refraction and diffraction; consequently, the model is limited to areas in which the assumption regarding the smallness of the bottom slope is valid.

A final point, relative to the use of a complex model in attempts to generate an estimate of a wave climate, involves the interaction between error terms in the wind field and the wave model. It is sometimes accepted in operational applications that the error in the estimation of winds over the water is so large as to preclude the need for a sophisticated wave model. The argument implies that, since the rms error in a wind estimate is, say, of the order of 10 knots, the wave model need only produce results with an rms error of ΔH , where ΔH is the range of wave heights possible with a 10-knot error. If the wave model contained only random error and zero bias, this might be a reasonable contention. However, some of the results of this paper clearly indicate that the models tested do not exhibit only random differences. Significant biases can arise in fetch-limited (and possibly time-limited) wave growth situations, if the proper growth relationships are not maintained. This error does not cancel out with the wind error, but rather it can introduce considerable bias into the wave estimates. The magnitude of this bias must be evaluated and the effect on the estimation of a wave climate considered before one can argue that the simpler model is adequate. Thus, all simplifying assumptions and model selection must be carefully evaluated in order to determine the magnitude of bias that they can introduce in climatic estimates. This magnitude should be roughly independent of the magnitude of wind error, if nonlinear effects due to the convolution of the two error distributions can be neglected. In a similar fashion, the bias in wind estimates should be isolated and evaluated. With four parameters (bias in wind estimates, bias in wave estimates, random error in wind estimates, random error in wave estimates) and the assumption that the

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errors are normally distributed, one can then compute the approximate inaccuracies due to using a wave hindcast model to obtain a wave climate. A simple comparison of the magnitude of random errors in wind estimates and the wave model alone cannot provide this information.

REFERENCES

Banner, M. L., and O. M. Phillips, 1974: On the incipient breaking of small scale waves. J. Fluid Mech., 65, 647-656.

Barnett, T. P., 1968: On the generation, dissipation, and prediction of wind waves. J. Geophys. Res., 73, 513-529.

Barnett, T. P., and A. J. Sutherland, 1968: A note on the over-shoot effect in wind-generated waves, J. Geophys. Res., 73, 6879-6885.

Barnett, T. P., and J. C. Wilkerson, 1967: On the generation of ocean wind waves as inferred from airborne radar measurements of fetch-limited spectra. J. Mar. Res., 25, 292-321.

Bunting, D. C., 1970: Evaluating forecasts of ocean-wave spectra. J. Geophys. Res., 75, 4131-4143.

Cardone, V. J., 1969: Specification of the wind distribution in the marine boundary layer for wave forecasting., Tech. Rept. 69-1, Geophys. Sci. Lab., New York University, 99 pp.

Collins, J. I., and T. W. Wier, 1971: Prediction of shallow water spectra, Final Tech. Rept. TETRAT P-71-164-2, Tetra Tech, Inc., Pasadena, Calif., pp.

DeLeonibus, P. S., and L. S. Simpson, 1972: Case study of duration-limited wave spectra observed at an open ocean tower. J. Geophys. Res., 77, 4555-4569.

DeLeonibus, P. S., L. S. Simpson and M. G. Mattie, 1974: Equilibrium range in wave spectra observed at an open-ocean tower., 79, 3041-3053.

Dexter, P. E., 1974: Tests on some programmed numerical wave forecast models. J. Phys. Oceanog., 4, 635-644.

Hasselmann, K., 1962: On the non-linear energy transfer in a gravity-wave spectrum - general theory. J. Fluid Mech., 12, part 1, 481-500.

Hasselmann, K., 1963a: On the non-linear energy transfer in a gravity-wave spectrum - conservation theorems - wave-particle analogy - irreversibility. J. Fluid Mech., 15, part 2, 273-281.

Hasselmann, K. 1963b: On the non-linear energy transfer in a gravity-wave spectrum - evaluation of the energy flux and swell-sea interaction for a Neuman spectrum. J. Fluid Mech., 15, part 3, 385-398.

Hasselmann, K., 1968: Weak interaction theory of ocean waves. In: Basic developments in fluid dynamics (Ed. M. Holt). New York, 2, 117-182.

Hasselmann, K., 1974: On the spectral dissipation of ocean waves due to white capping. Bound.-Layer Meteor., 6, 107-127.

Hasselmann, K., and J. I. Collins, 1968: Spectral dissipation of finite-depth gravity waves due to turbulent bottom friction, J. Mar. Res., 26, 1-12.

Hasselmann, K. et. al. 1973: Measurements of wind-wave growth and swell decay during the Joint North Sea Wave Project JONSWAP. Deutsches Hydrographisches Institut, Hamburg Germany, 95 pp.

Hasselmann, K., D. B. Ross, P. Muller and W. Sell, 1976: A parametric wave prediction model, J. Phys. Oceanogr., 6, 200-228.

Inoue, T., 1967: On the growth of the spectrum of a wind-generated sea according to a modified Miles-Phillips mechanism and its application to wave forecasting. Tech. Rept. 67-5, Geophys. Sci. Lab. New York University, 74 pp.

Kitaigorodskii, S. A., 1962: Application of the theory of similarity to the analysis of wind-generated wave motion as a stochastic process. Bull. Acad. Sci. USSR Geophys. Ser. No. 1, 73, 105-117.

Laxanoff, S. M., and N. M. Stevenson, 1975: An evaluation of a hemispheric operational wave spectral model. U. S. Navy Fleet Numerical Weather Central Tech. Note 75-3, 103 pp.

Lazanoff, S. M., N. M. Stevenson and V. J. Cardone, 1973: A Mediterranean Sea wave spectral model. U. S. Navy Fleet Numerical Weather Central Tech. Note 73-1, 93 pp.

Liu, P. C., 1971: Normalized and equilibrium spectra of wind waves in Lake Michigan, J. Phys. Oceanog., 1, 249-257.

Miles, J. W., 1957: On the generation of surface waves by shear flows. J. Fluid Mech., 3, 185-204.

Miles J. W., 1960: On the generation of surface waves by turbulent shear flows. J. Fluid Mech., 7, 469-478.

Mitsuyasu, H., 1968a: A note on the nonlinear energy transfer in the spectrum of wind-generated waves. Rept. Res. Inst. for Appl. Mech., 16, 251-264.

Mitsuyasu, H., 1968b: On the growth of wind-generated waves (I), Rept. Res. Inst. for Appl. Mech., Kyushu University, Fukuoka, Japan, 16, 459-482.

Mitsuyasu, H., 1973: The one-dimensional wave spectra at limited fetch. Rept. Res. Inst. for Appl. Mech., 20, 37-53.

Mitsuyasu, H. and Rikiishi, 1975: On the growth of duration limited wave spectra. Rept. Res. Inst. for Appl. Mech., 23, 31-60.

Phillips, O. M., 1957: On the generation of waves by turbulent wind. J. Fluid Mech., 2, 417-445.

Phillips, O. M., 1958: The equilibrium range in the spectrum of wind-generated waves. J. Fluid Mech., 4, 426-434.

Phillips, O. M., 1966: The Dynamics of the Upper Ocean. Cambridge University Press, 261 pp.

Pierson, W. J., Jr., and L. Moskoivitz, 1964: A proposed spectral form for fully developed wind seas based on the similarity theory of S. A. Kitaigorodskii. J. Geophys. Res., 69, 5181-5190.

Pierson, W. J. and R. A. Stacy: 1973, The elevation, slope, and curvature spectra of a wind-roughened sea surface, NASA Contractor Report NASA CR-2247 under contract NAS1-10090, 128 p.

Resio, D. T. and C. L. Vincent, 1976: Design wave information for the Great Lakes: Report 2 - Lake Ontario. Tech. Rept. H-76-1, U. S. Army Waterways Experiment Station, Vicksburg, MS., 148 pp.

Ross, D. B., V. J. Cardone and J. W. Conway, Jr., 1970: Laser and microwave observations of sea-surface condition for fetch-limited 17- to 25 m/s winds. IEEE Trans. on Gevscience Electronics, GE-8, 326-336.

Salfi, R. E., 1974: Operational computer based spectral wave specification and forecasting models. The University Inst. of Oceanog. of the City Univ. of New York, prepared for SPOC Group of the Nat'l Environmental Sattelite Service, NOAA, under Grant No. 04-4-158-11, 130 pp.

Sanders, J. W., 1976: A growth-stage scaling model for the wind-driven sea, Dt. Hydrogr. Z. Heft4, pp. 136-161.

Schule, J. J., Jr., L. S. Simpson and P. S. DeLeonibus, 1971: A study of fetch-limited wave spectra with an airborne laser., J. Geophys. Res., 78, 4160-4171.

Snodgrass, F. E., G. W. Groves, K. F. Hasselmann, G. R. Miller, W. H. Munk and W. H. Powers, 1965: Propagation of ocean swell across the Pacific. Phil. Trans. Roy. Soc. (A), 259, 431-497.

Sutherland, A. J., 1968: Growth of spectral components in a wind-generated wave train. J. Fluid Mech., 33, 545-560.

Sverdrup, H. U. and W. H. Munk: 1947, Wind, sea, and swell: Theory of relations for forecasting, H. O. Pub. No. 601, U. S. Navy Hydrographic Office, Washington, D. C., 44 p.

NOTATION

a	Phillips' equilibrium coefficient
b_1, b_2	Coefficients relating Phillips' "equilibrium constant" to nondimensional wave height
c	Phase velocity
c_g	Group velocity of waves with wave number modulus k
$c_{g_{max}}$	Group velocity of lowest frequency included in the calculations
D	Fetch
$\overline{D}$	Nondimensional fetch
E_{max}	Maximum attainable energy density for a frequency f in the equilibrium range
E_o	Total energy within the spectrum
$\overline{E}, \tilde{E}$	Normalized functions of relative frequency
f	Frequency
$F(f, \theta)$	Frequency-direction representation of spectral energy density
$F'(k_1, k_2)$	Wave number representation of spectral energy density
$F^n_{ijk\ell}$	Energy density associated with the wave particle located at grid point (i,j) in $\underline{x}$-space and point (k,ℓ) in $\underline{k}$-space.
F_1, F_2	Energy densities at the beginning and end of the time step, respectively
f_c	Frequency at which the spectrum reaches the equilbrium range
f_h	High frequency cut off in numerical model
f_m	Frequency of the spectral peak
$\overline{f_m}$	Nondimensional spectral peak frequency
g	Gravity
ΔH	Range of wave heights possible with a 10-knot rms error
$\overline{H}$	Nondimensional wave height

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i,j	Location indices of the wave particle (in $\underline{x}$-space)
$\underline{k}$	Wave number vector
k_i	i^{th} component of $\underline{k}$
k_1,k_2	Orthogonal components of $\underline{k}$
k	Wave frequency index associated with the wave particle (when used as subscript)
ℓ	Direction index of propagation of wave particle
n	Time step index
n_1,n_2	Emperical constants for form of β source term
R	Function used to provide an off-diagonal interpolation between grid points
S	Sum of all sources and sinks of wave energy
S_B	Energy sink due to white capping
t	Time in a two-dimensional wave number spectrum
$\bar{t}$	Nondimensional time
Δt	Time increment
$\Delta t'$	Time increment required for wave particle to travel from one grid point to another
u	Wind velocity at some reference level
u_*	Friction velocity of the wind
$\underline{x}$	Spatial location of wave spectrum
x_i	i^{th} component of $\underline{x}$
x_1,x_2	Orthogonal components of $\underline{x}$
Δx	Grid spacing
α,β	Sources of energy input from the winds (Phillips and Miles mechanisms, respectively)
Γ,τ	Wave-wave interaction source terms
$\underline{\delta},\underline{\lambda},\underline{\lambda}'$	Subscripts used to locate reference values of F in wave growth-decay equation

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θ Direction of wave propagation

μ Energy loss due to wave breaking

ϕ Loss of energy due to interaction with bottom

δ' Sum of exponential source terms in radiative transfer equation

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